Enrollment No: _____ **Exam Seat No:**_____

C.U.SHAH UNIVERSITY
Winter Examination-2015

Subject Name : Engineering Mathematics IV

Subject Code : 4TE04EMT1 **Branch :** Auto/Mech/Civil/EEE/EE/IC/EC

Semester : 4 **Date :** 18/11/2015 **Time :** 2:30 To 5:30 **Marks :** 70

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions: (2 Marks each)** **(14)**

a) To interpolate the value of a function near the beginning of tabular values, Stirling's interpolation formula can be used. True or False? Justify

b) Prove that $\mu = \frac{1}{2}\left(E^{1/2} + E^{-1/2}\right)$

c) In Numerical differentiation $\left(\frac{d^2y}{dx^2}\right)_{x=x_0}$ = _____

d) State the condition of convergence in Gauss-Seidal method.

e) To evaluate definite integral with even number of sub intervals, Simpson's $1/3$ rule can be used. True or False? Justify.

f) Write the Cauchy-Riemann equations in polar form.

g) Check whether the vector $\vec{V} = (x-y)\hat{i} + (y-z)\hat{j} - (x-z)\hat{k}$ is solenoidal.

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

A Using Lagrange's interpolation formula, find y at x = 6

x	3	7	9	11
y	168	120	72	63

B Find the Fourier transform of $e^{-x^2a^2}$, $a>0$ and deduce that $F\left(e^{-\frac{\lambda^2}{2}}\right)=e^{-\frac{\lambda^2}{2}}$

Q-3 **Attempt all questions** **(14)**

A The population of a certain town is given below. Find the rate of growth of the population in 1931 to 1971.

Year (x)	1931	1941	1951	1961	1971
Population (y) In thousands	40.62	60.80	79.95	103.56	132.65

B Define Harmonic function. Show that the function $\frac{1}{2}\log(x^2+y^2)$ is harmonic and determine its conjugate.

Q-4 **Attempt all questions** **(14)**

A Using Runge-Kutta method of fourth order ,find y(0.2) by taking h=0.1 if

$$\frac{dy}{dx}=x+y^2 \quad , \quad y(0)=1$$

B Find the analytic function $f(z)=u+iv,$ whose real part is $e^x(x\cos y-y\sin y).$ Also find the corresponding imaginary part.

Q-5 **Attempt all questions** **(14)**

A Using Taylor's Series method, solve

$$\frac{dy}{dx}=x^2y-1 \quad , \quad y(0)=1 \ for \ x=0.1 \ and \ x=0.2$$

B Verify Stokes's theorem for $\vec{F}=\left(x^2+y^2\right)\hat{i}-2xy\,\hat{j}$ over the rectangle bounded by the lines $x=\pm a\,, y=0, y=b.$

Q-6 **Attempt all questions** **(14)**

A Solve the following system of equations by Gauss elimination method

3x + 4y – z = 8 , –2x + y + z = 3 , x + 2y – z = 2

B Verify Green's theorem in the plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region defined by $y = \sqrt{x}, y = x^2$

Q-7 **Attempt all questions** **(14)**

A Solve the following system of equations by Gauss – Seidel method correct to three decimal places.

$10x + 2y + z = 9$, $2x + 20y - 2z = -44$, $-2x + 3y + 10z = 22$

B A particle moves along the curve $x = 2t^2,\ y = t^2 - 4t,\ z = 3t - 5$, where t is the time. Find the components of its velocity and acceleration at time $t = 1$ in the direction $\vec{i} - 3\vec{j} + 2\vec{k}$.

Q-8 **Attempt all questions** **(14)**

A Evaluate $\int_0^1 \frac{dx}{1+x}$ taking 7 ordinates by applying Simpson's $\frac{3}{8}$ rule. Also compare your answer with actual value obtained by usual integration.

B Using Fourier integral, Prove that $\int_0^\infty \frac{\cos \omega x + \omega \sin \omega x}{1+\omega^2} d\omega = \begin{cases} 0 & , x < 0 \\ \frac{\pi}{2} & , x = 0 \\ \pi e^{-x} & , x > 0 \end{cases}$